THE CONTRIBUTIONS OF GEODESY TO GEOGRAPHY

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The determination of the shape and size of the earth and the location of places on its surface, referred to selected fundamental planes, are the geodesist's principal contributions to geography.

The earliest geographers must have found it very difficult to carry on their operations, owing to lack of knowledge of the shape and size of the earth and of the true relative positions of the areas under investigation.

A very good value for the size of the earth was obtained from the measurement by triangulation of a meridional arc of about 8½ degrees extending north and south of Paris by Domenico and Jacques Cassini between the years 1683 and 1716.[1] But the fact that the earth's mean figure is an oblate spheroid was only discovered as a result of the observations made in Peru and in Lapland. This work was begun in 1735.[2]

The first near approach to the actual figure of the earth resulted from triangulation done in the last decades of the eighteenth century to connect the observatories of Paris and Greenwich and to determine the length of the earth's meridian quadrant. For the latter purpose an arc of the meridian of nearly ten degrees was measured in France. One ten-millionth of the resulting length was adopted as the standard of length (the meter). Any other length could have been selected, as the standard for the meter bears no such exact relation to a quadrant, as later and more accurate data show.[3]

Geodesists were very active during the nineteenth century, and will be for some time to come, in making geodetic measurements to determine the mean figure of the earth with greater and greater precision. There comes a time, for any given area, when it is useless to add more geodetic data for the purpose of obtaining a more nearly exact mean figure, for there are constant or systematic errors present in the data the effect of which is probably much greater than that of the accidental errors.

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[1] These operations are described by the younger Cassini, Jacques, in the *Mémoires de l'Académie Royale des Sciences*, "De la grandeur de la terre et de sa figure," Paris, 1718, and in another work, "Traité de la grandeur et de la figure de la terre," Paris, 1720, and Amsterdam, 1723.

[2] Pierre-Louis-Moreau de Maupertuis: La figure de la terre déterminée par Messieurs de l'Académie Royale des Sciences qui ont mesuré le degré du méridien au cercle polaire, *Mémoires de l'Académie Royale des Sciences*, Paris, 1737. Pierre Bouguer: La figure de la terre déterminée par les observations de Messieurs Bouguer et de la Condamine de l'Académie Royale des Sciences, envoyés par ordre du Roy au Pérou, pour observer aux environs de l'Equateur, Paris, 1749. Charles Marie de la Condamine: Mesure des trois premier degrés du meridién dans l'hémisphère austral tirée des observations de Mrs. de l'Académie Royale des Sciences, envoyés par le Roi sous l'Equateur, Paris, 1751.

[3] Modern measures give the length of a quadrant of a meridian greater by about 2,000 meters than the intended 10,000,000 meters.

But there is much to be gained by extending geodetic surveys to new areas and especially to new continents. (By geodetic survey is meant, here, triangulation and observations at connected astronomic stations.) Nearly all of the values of the earth's figures now available are the result of geodetic measurements in the northern hemisphere, and the measurements in that hemisphere, from which the earth's figure has been derived, have been confined to India, Europe, and the United States. We may hope to get, before long, values for the figure of the earth from geodetic operations in South America, Africa, and Australia. It is expected that the mean figures resulting from accurate and extensive geodetic data in those continents will agree closely with the figures obtained from continents in the northern hemisphere. The geodetic surveys of the several nations on each continent should be connected and the reductions made on one spheroid and referred to a single initial position for each continent. Should this be done we shall be able eventually to compute a mean figure of the earth which will be of such great precision that it will satisfy the most exacting demands of science.

The Geoid

Coincident with the extension of geodetic surveys there will be carried on the computation of the geoid. The surface of the geoid is probably so complex in shape that the work necessary to define it will have to be continued long after the satisfactory spheroid has been determined.

The geoid may be defined as that surface which coincides with the surface of the sea at rest. We can imagine an extension into the continents of an intricate network of sea-level canals. Then the surface of the oceans and of the water in the canals would define the surface of the geoid. At some points, probably not exactly at the sea shore, the mean surface of the earth—the spheroid—would intersect the actual sea surface, the geoid. Under the coastal plains the geoid would be slightly above the spheroid, while under great mountain ranges the geoid would be far above the spheroid, possibly as much as one hundred meters. Over the oceans the geoid would be under the spheroid surface by amounts depending upon the depths of the water.

There is only one way to determine accurately the size of the earth, and that is by measurement on the continents of the lengths of arcs connecting points where the astronomic latitude and longitude have been determined. The measurements of such arcs may be direct, or they may be by means of triangulation. The earliest measurements were by the former method, but, with the introduction of accurately graduated circles and the application of wires in the eyepieces of telescopes, the indirect method came into general use.

At frequent intervals, in triangulation, the sides of some of the triangles in the scheme are accurately measured, in order to control the lengths. At the present time, this is done almost exclusively with nickel-steel (invar)

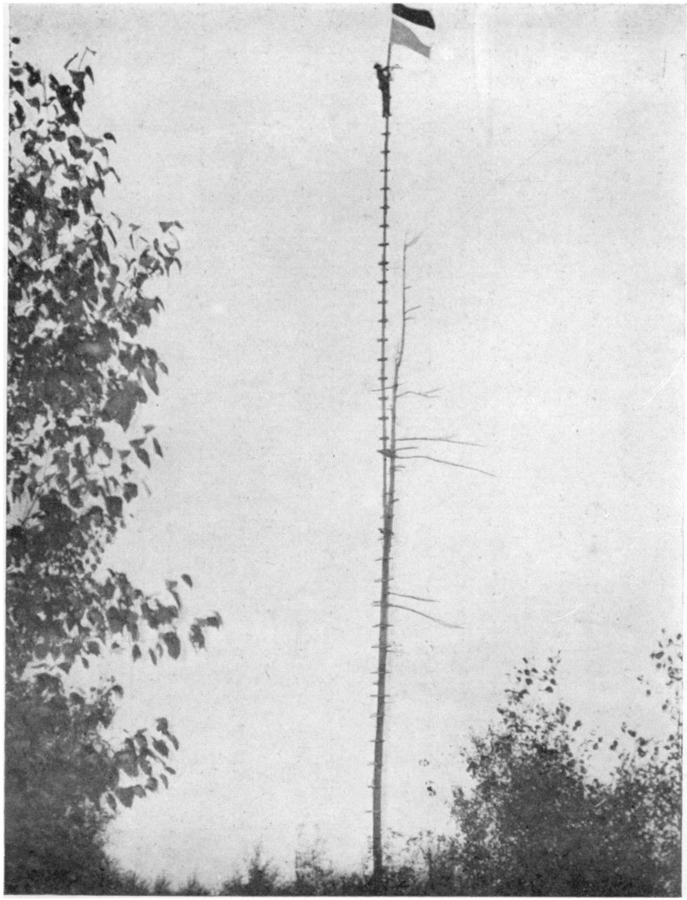

FIG. 1—Triangulation reconnaissance signal of the U. S. Coast and Geodetic Survey, used in the selection of stations.

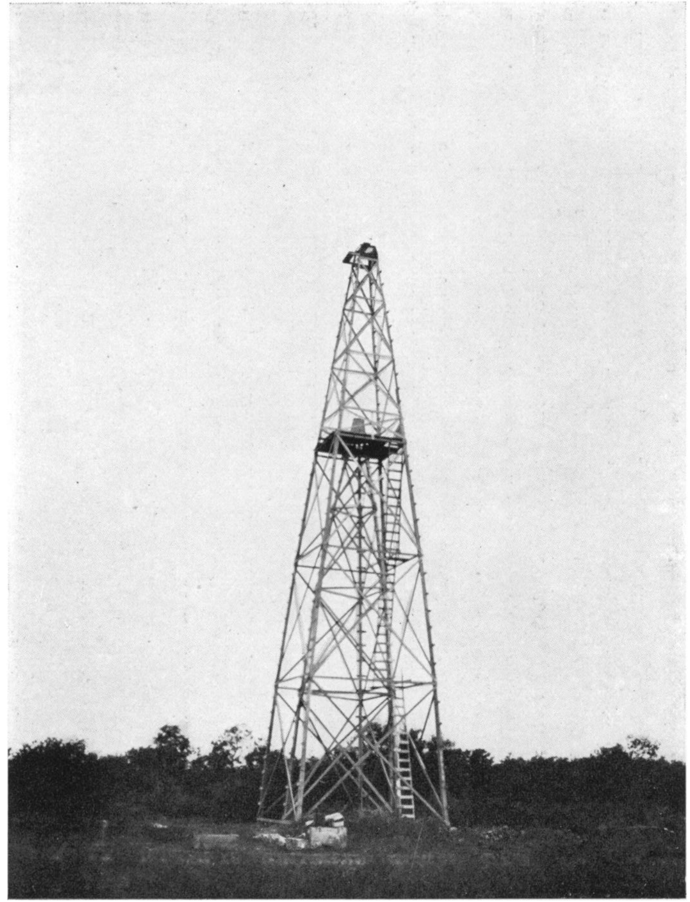

FIG. 2—Triangulation observing tower of the U. S. Coast and Geodetic Survey, used to raise the theodolite above obstructions.

tapes or wires. The probable accidental error of a measured length is seldom greater than about 1 in 1,000,000. The constant error in such a measurement may be as great as 1 in 300,000. This accuracy is, however, far greater than that of the lengths of the triangle sides, as computed through the chain of triangles. The uncertainty of any one line between bases is about 1 in 100,000, on an average. A long arc, say one across a continent, can be measured with greater accuracy than that, for even the systematic and constant errors of the various sections of the arc would probably act as accidental errors and the greater portion of their effect would be eliminated.

The observations for latitude, longitude, and azimuth, or direction, are made on the stars; and in the most refined work a correction is made for the variation of the pole.

One might think that the determination of the figure and size of the earth is a very simple process, consisting merely of obtaining by accurate observations the accurate astronomic latitudes of several points on a meridian and then measuring accurately the linear distances between them. Three such points being sufficient to obtain the equation of the ellipse formed by the intersection of the meridional plane and the spheroid, the shape and size of the earth would be known. This would be true if the spheroid and the geoid coincided throughout, but, as stated above, they do not do so. The plumb line, to which all astronomic observations are referred, is, at each point, normal to the geoid, which is a very irregular surface, and, therefore, very many astronomic stations must be established and used. The greater part of each of the differences between the observed astronomic position and the position referred to an adopted smooth mean surface, must be treated as an accidental error in the computation of the figure of the earth. These differences, called deflections of the vertical, also station errors, reach a maximum value of about twenty-five seconds of arc (within the area of the United States), which is nearly one half mile. In the island of Porto Rico, the relative deflection between two astronomic stations, one at Ponce and the other at San Juan, was 56 seconds of arc, or about one mile.

The shape, but not the size, of the earth may be determined from the observed value of gravity at stations widely distributed in latitude. But here again a few stations are not sufficient, for the change in the value of gravity with the latitude and with the elevation above sea level does not exactly follow any regular law, owing to the disturbing influences of masses above sea level and the deficiency of mass in the oceans.

It is evident that the difference between the theoretical and the observed values of gravity and the deflections of the plumb line (which, as stated above, are the differences between the observed and the theoretical astronomic positions) are due to the disturbing influences of the topography and the effect of deviations from the normal densities in the earth's crust.

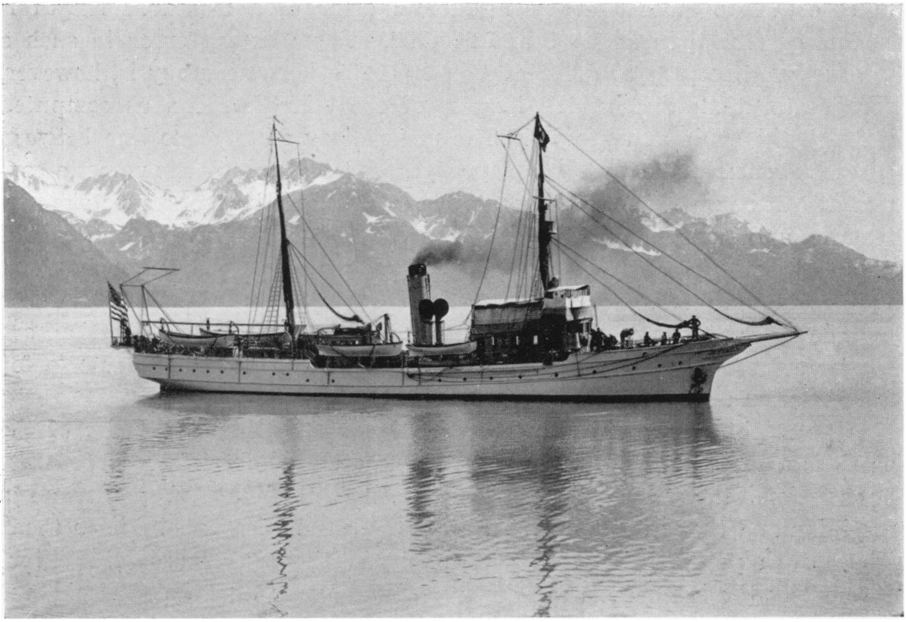

FIG. 3.

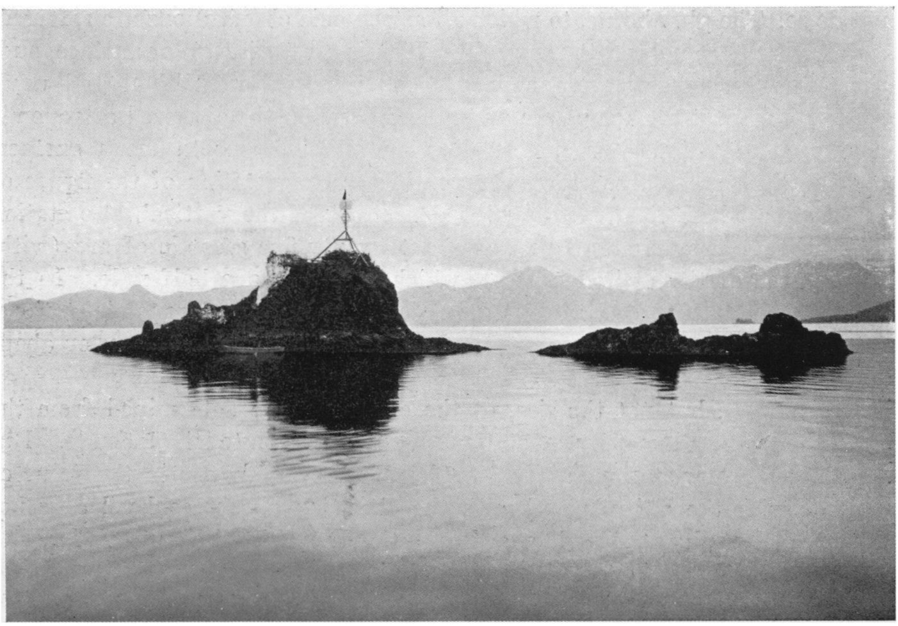

FIG. 4.

FIG. 3—The *Explorer*, a vessel of the U. S. Coast and Geodetic Survey, used along the Pacific coast and the coast of Alaska.

FIG. 4—Triangulation station of the U. S. Coast and Geodetic Survey on a rocky island in Prince William Sound, Alaska.

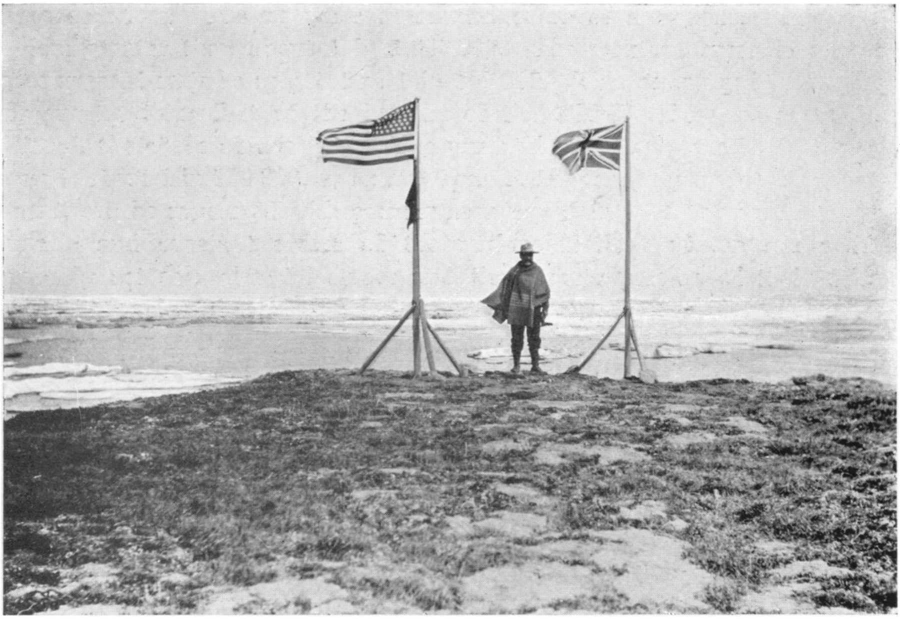

FIG. 5.

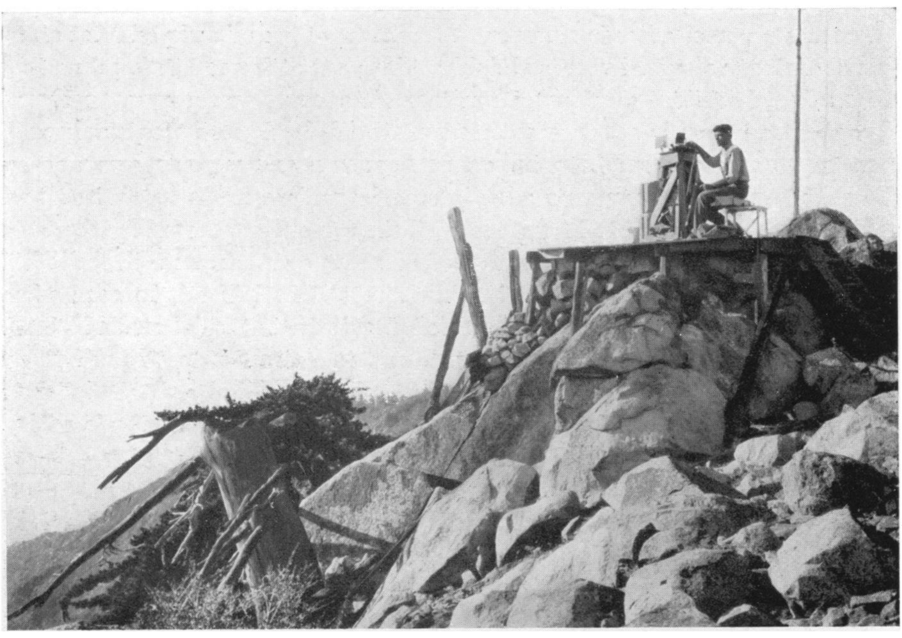

FIG. 6.

FIG. 5—Triangulation station of the International Boundary Commissions at the northern extremity of the boundary between Alaska and Canada. The Arctic Ocean in the background.

FIG. 6—Heliographer at work on a triangulation station of the U. S. Coast and Geodetic Survey, Cuyamaca Mountain, southern California. This heliograph was observed from station American, distant 108 miles. (Photo by J. Smeaton Chase.)

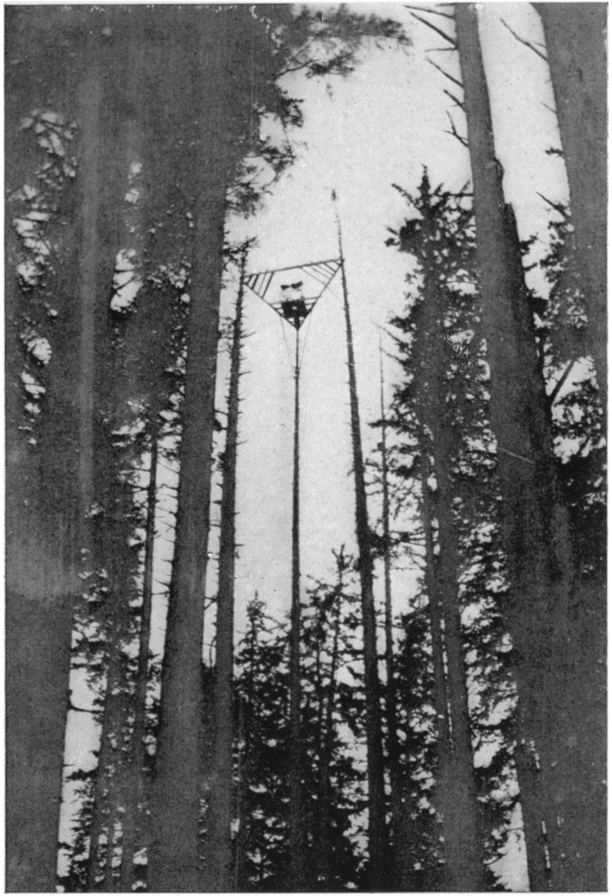

FIG. 7—Structure at La Push triangulation station, near the coast of Washington, made from three standing trees, for use in elevating the theodolite and triangulation observer above the surrounding timber. The instrument support is 187 feet above the ground.

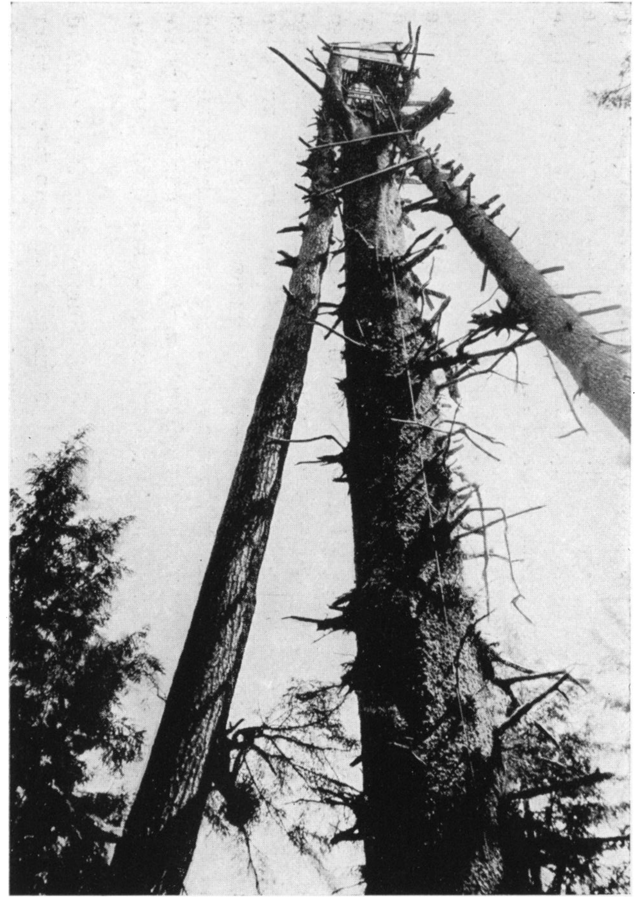

FIG. 8—Instrument support and observing platform at triangulation station Hoh, near the coast of Washington, made by throwing three trees together. Height above ground, 119 feet.

The term "topography" is applied to the visible land masses and the deficiency of mass in the oceans. But, even when the attractions of the topography are applied as corrections, the differences, which may still be large, would be generally of the opposite sign.

Isostasy

About sixty years ago, Archdeacon Pratt of Calcutta arrived at the conclusion, from a study of the deflections in India, that there must be a deficiency of mass under the Himalaya Mountains and that the deficiency extended to a limited depth.[4] The announcement of this theory marked an epoch in geodesy. From time to time, writers in different countries have elaborated on the mere statement of Pratt.[5] But it was Hayford who gave this theory a quantitative expression when, as a member of the U. S. Coast and Geodetic Survey, he corrected the astronomic latitudes, longitudes, and azimuths in the United States for the effect of topography and its negative equivalent, called "isostatic compensation," when making two determinations of the figure of the earth.[6]

Several reports by the Coast and Geodetic Survey give the results of investigations based upon the subject of isostasy,[7] and a number of other articles have appeared in recent years, notably those by Dutton, Helmert, Barrell, Becker, Hecker, and Gilbert.[8]

If the earth were composed of homogeneous material, or if at all points at any given depth the density were the same, the earth's surface, due to its own rotation and the force of gravitation, would be very nearly a true

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[4] John Henry Pratt: On the Deflection of the Plumb-line in India, caused by the attraction of the Himalaya Mountains and of the elevated regions beyond; and its modification by the compensating effect of a deficiency of matter below the mountain mass; also, On the Influence of the Ocean on the Plumb-line in India, *Philos. Trans.*, Vol. 149, 1859, London.

[5] A. R. Clarke: Geodesy, pp. 96 and 350. H. A. Faye: Sur la réduction des observations du pendule au niveau de la mer, *Comptes-Rendus de l'Acad. des Sci.*, Vol. 90, 1880, Paris.

[6] J. F. Hayford: The Figure of the Earth and Isostasy from Measurements in the United States, U. S. Coast and Geodetic Survey, Washington, 1909; *idem:* Supplementary Investigation in 1909 of the Figure of the Earth and Isostasy, U. S. Coast and Geodetic Survey, 1910.

[7] In addition to the two mentioned in the preceding footnote: O. H. Tittmann and J. F. Hayford: Geodetic Operations in the United States, 1903-06, U. S. Coast and Geodetic Survey, 1906; J. F. Hayford and William Bowie: The Effect of Topography and Isostatic Compensation upon the Intensity of Gravity, *U. S. Coast and Geodetic Surv. Special Publ. No. 10*, 1912; William Bowie: Effect of Topography and Isostatic Compensation upon the Intensity of Gravity (Second Paper), *U. S. Coast and Geodetic Surv. Special Publ. No. 12*, 1912.

[8] C. E. Dutton: On Some of the Greater Problems of Physical Geology, *Bull. Philos. Soc. of Washington*, Vol. 11, 1888-91, pp. 51-64.

R. F. Helmert: Die Schwerkraft und die Massenverteilungen der Erde, in "Encyclopädie der Mathematischen Wissenschaften," Band VI, 1 B, Heft 2, Leipzig; Unvollkommenheiten im Gleichgewichtszustande der Erdkruste, *Sitzungsber. der Kgl. Preussischen Akad. der Wiss.*, Vol. 44, 1908, Berlin.

Joseph Barrell: The Strength of the Earth's Crust (a series of articles), *Journ. of Geol.*, Vols. 22 and 23, 1914 and 1915.

G. F. Becker: Isostasy and Radio-activity, *Bull. Geol. Soc. of America*, Vol. 26, 1915, pp. 171-204.

Oskar Hecker: Bestimmung der Schwerkraft auf dem Atlantischen Ozean, sowie in Rio de Janeiro, Lissabon und Madrid, Kgl. Preussiches geodät. Institut, Berlin, 1903; Bestimmung der Schwerkraft auf dem Schwarzen Meere und an dessen Küste, sowie neue Ausgleichung der Schwerkraftsmessungen auf dem Atlantischen, Indischen, und Grossen Ozean, Zentralbureau der Internationalen Erdmessung, Berlin, 1910.

G. K. Gilbert: Interpretation of Anomalies of Gravity, *U. S. Geol. Surv. Prof. Paper 85-C*, 1913.

ellipsoid of revolution. These conditions as to the distribution of density do not apply universally. The earth's surface is very irregular, as is shown by the existence of continents and oceans.

Geodetic observations and their discussion show conclusively that the continents and oceans are not held in place by the strength of the earth's crust but exist and are maintained by a deficiency and excess of density, respectively, under them in the outer portions of the earth's volume. The investigations of the Survey show that at all places at a depth of about 120 kilometers (75 miles) below sea level there is an approximate condition of equilibrium as to pressures. This condition of approximate equilibrium has been given the name of *isostasy.* It has been proved by recent investigations that the area of the United States as a whole is in a state of isostasy to a very high degree of completeness. Whether small sections of this area are in such a condition must be a subject for further research. The extent of an area for which the topography may not be compensated is a question which should be solved as soon as practicable, for the result will be of great value to many branches of science.

The question may be raised, Why should a geodesist be interested in the question of the variations in the density of the materials in the earth's crust? The answer is that a knowledge of the variations of density enables him to apply corrections to the deflection of the plumb line and the observed values of the intensity of gravity, and thus obtain from the results very much more accurate values for the earth's figure.

While isostasy has been from the first a subject of great importance to the geodesist, it has become an even greater one to the geologist and seismologist. Many geologic hypotheses and theories must be modified to conform to the facts deduced from the results as to the variations of density in the outer portion of the earth, obtained from geodetic observations.

Triangulation

Thus far we have considered only the value of geodetic measurements in connection with the determination of the figure of the earth. They have also a very practical and immediate value in determining the positions of the topographic features with relation to the fundamental planes. The generally adopted system of co-ordinates is spherical and is referred to the plane of the equator for latitudes, the plane of the meridian through the observatory at Greenwich, England, for longitude, and the mean sea surface for elevation.

Owing to the station errors or deflections of the vertical at astronomic stations it is not possible to obtain the correct relative position of different places on the earth's surface by astronomic observations alone. This was strikingly shown in Porto Rico. The Spanish charts of this island were based upon astronomical stations at San Juan and Ponce on the northern and southern sides of the island, respectively. When the United States

became the possessor of this island, it was decided that a strong triangulation should be carried across the island from San Juan to Ponce, as the fundamental control from which new surveys should expand. The distance between the two places by triangulation was found to be about one mile shorter than the distance given by the two astronomic determinations. The triangulation across the island is subject to an actual error not greater than ten meters as a maximum.

The trouble was at the astronomic stations but was not due to errors in the observations. There was a relative deflection of the vertical of about 56″ of arc, due to the attraction of the island mass and the repelling force,

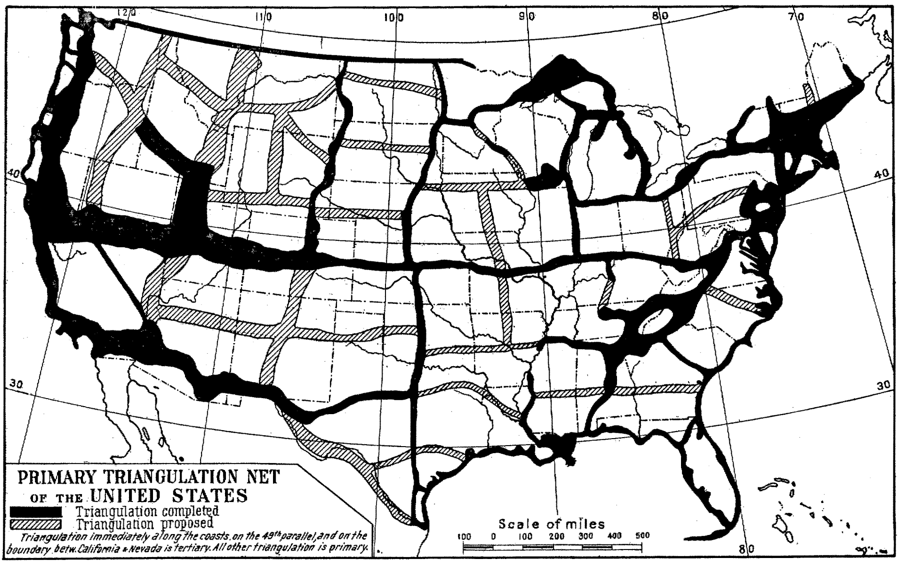

FIG. 9—Map showing the network of arcs of primary triangulation in the United States. (From Pl. 2, *U. S. Coast and Geodetic Survey Special Publ. No. 23*). Scale, 1:41,000,000.

The solid bands show the completed work and the shaded bands the work which will be done in the near future.

or what might be called a lack of attraction, of the vast volumes of the Atlantic Ocean to the north and the Caribbean Sea to the south.

The same phenomenon has been observed in the interior of the United States, where there are astronomic stations short distances apart and on opposite sides of a large mountain range.

It is evident from the above that an accurate map cannot be made over a large area where each of the separate sections is based on the astronomic position of the starting point of each of the various surveys. There will be overlaps and gaps which cannot be adjusted in a satisfactory manner. This difficulty is overcome by making a triangulation ahead of the surveys, for the connected scheme of triangulation will give the correct relative positions of the several stations. To obtain the most probable absolute positions

on the earth's surface of these stations a mean position for latitude and longitude is obtained by connecting into the triangulation scheme many astronomic stations. It is assumed that in a large area there are as many positive as negative deflections, and for a country the size of the United States this is very nearly true.

In the United States a mean astronomic position was adopted in 1901 and called the United States Standard Datum. Several years ago this datum was also adopted by Canada and by Mexico and then its designation was changed to that of the *North American Datum.* The triangulation of Alaska will soon be connected to that of the United States and Canada and then maps of its area will also be based on the continental datum. When the whole mapped areas of Alaska, Canada, the United States, and Mexico are based upon a connected scheme of triangulation, which will no doubt be done within the next few decades, this continent will be in a very enviable position, as far as control of maps is concerned. It should be said here that for the control of large areas, triangulation of the highest order must be extended in a network of long arcs. Lower-grade triangulation can be used to fill in the intermediate areas for the immediate control of the detailed topographic surveying and the map making.

Precise Leveling

In nearly all geographic work a knowledge of the elevation of the area under investigation is also necessary.

In rough work, such as topographic reconnaissance and exploration the barometer (mercurial or aneroid) gives results which are satisfactory; in fact, this is the only instrument adapted to such work. But the atmospheric pressure at any given place is so variable that the elevations obtained with the barometer are much in error, even when readings are made simultaneously at base stations, unless the line of base stations is carried along in very short steps.

For all accurate topographic work it is necessary to have leveling done with the wye or spirit level. This instrument and the results obtained with it are no doubt familiar to the reader. There are many such instruments in use, but the types of most interest to us are what are termed precise levels. With these, lines of levels may be extended thousands of miles with no appreciable error so far as the purposes of geography are concerned.

Eighty-three per cent of the precise leveling done in the United States has errors of closures of circuits which are not more than 1.57 thousandths of a foot per mile. As the precise level net is made up of many circuits, it is reasonably certain that the absolute error in the elevation of any precise level bench mark in the interior of this country is not more than one and one-half feet and the probable error is considerably less than that. It is necessary to cover the country with a network of precise leveling in order that errors of the elevations carried inland may not accumulate to a trouble-

some extent. This is readily understood when we consider the great distances from the coasts of this country to interior places. The error in ordinary leveling between any two contiguous bench marks might be small, but, when such leveling is carried from the Atlantic to a point in Minnesota, for instance, and to the same point from the Pacific Coast, the difference between the two elevations obtained for the same point might be many feet. Such an error would be a source of great confusion to the surveyor and map maker.

We have seen that the province of geodesy in geography is to furnish the correct dimensions of the earth, to determine approximately the distribution of material in the outer portions of the earth, and to furnish the correct positions and elevations on the earth's surface of the starting points for surveys and maps. Realizing the great importance of geodetic surveys and investigations, nearly all of the nations of the world have organizations for the purpose of carrying on this work; and to co-ordinate the results and to undertake the international phases of this important subject there is an International Geodetic Association, in which more than twenty nations are represented.